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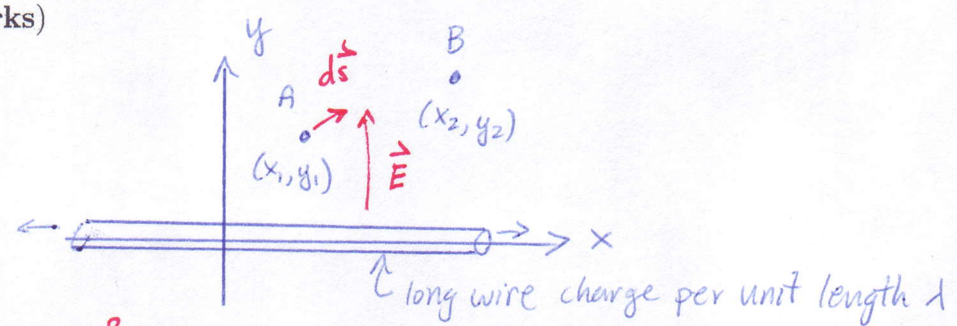
1. The electric field due a long wire with charge per unit length λ is:

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

Suppose the wire lies along the x-axis. What is the difference in electric potential between points A and B? (**4 marks**)



$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s} \quad (1)$$

In xy-plane can write $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 y}\hat{j}$ where y is dist. from wire.

$$\vec{E}\cdot d\vec{s} = \frac{\lambda}{2\pi\varepsilon_0}\frac{dy}{y} \quad (1)$$

Only displacement parallel to $\vec{E}$ results in change to potential.

$$\therefore \Delta V = -\int_{y_1}^{y_2} \frac{\lambda}{2\pi\varepsilon_0}\frac{dy}{y} = -\frac{\lambda}{2\pi\varepsilon_0}\int_{y_1}^{y_2}\frac{dy}{y} = -\frac{\lambda}{2\pi\varepsilon_0}\ln y\Big|_{y_1}^{y_2}$$

$$\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\left(\ln y_2 - \ln y_1\right)$$

$$\boxed{\Delta V = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{y_2}{y_1}\right)} \quad (2)$$

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3. For the potential $V = 3x^2 - 6xy$, what is the corresponding electric field at the point $(x, y) = (2, 2)$? **(3 marks)**

① $E_x = -\frac{dV}{dx} = -(6x - 6y)$

$\vec{E} = E_x \hat{i} + E_y \hat{j}$

① $E_y = -\frac{dV}{dy} = -(-6x)$

@ (2, 2) $E_x = -(12-12) = 0$

$E_y = -(-12) = +12$

$\therefore \boxed{\vec{E} = 12\hat{j}}$ ①

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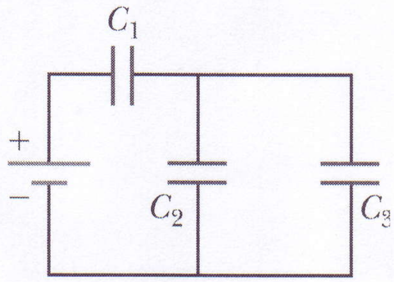
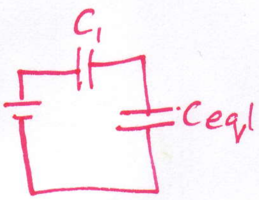
1. (a) Three capacitors are connected to a battery as shown in the figure. Their capacitances are $C_1 = 3C$, $C_2 = C$, and $C_3 = 5C$. (a) What is the equivalent capacitance of this set of capacitors? (**2 marks**)

(b) Find the charge stored on each capacitor in terms of C and the potential difference ΔV supplied by the battery. (**3 marks**)

(c) Assume that C_3 is increased, how does the charge stored by each capacitor change (increase or decrease)? (**3 marks**)

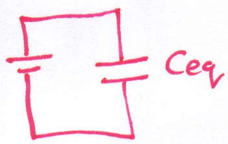


(a)



$C_{eq1} = C_2 + C_3 = 6C$ (1)

$\Rightarrow$



$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{eq1}}$

$= \frac{1}{3C} + \frac{1}{6C} = \frac{3}{6C}$

$\therefore C_{eq} = 2C$ (1)

(b) $C = \frac{q}{\Delta V}$

$\therefore q_{eq} = C_{eq} \Delta V$
$= 2C\Delta V$

capacitors in series have same charge

$\therefore$ $q_1 = 2C\Delta V$ (1)

$q_{eq1} = 2C\Delta V$

$\Delta V_{eq1} = \frac{q_{eq1}}{C_{eq1}} = \frac{2C\Delta V}{6C} = \frac{\Delta V}{3}$

capacitors in parallel have same potential.

$\therefore \Delta V_2 = \Delta V_3 = \Delta V/3$

$q_2 = C_2 \Delta V_2 = \frac{C\Delta V}{3}$ (1)

$q_3 = C_3 \Delta V_3 = \frac{5C\Delta V}{3}$ (1)

(c) suppose C_3 very large.

Then, $C_{eq1} \approx C_3$

$C_{eq} \approx C_1$

$q_1 = C_{eq} \Delta V \approx C_1 \Delta V$
$\approx 3C\Delta V$

$\therefore q_1$ increases (1)

Parallel combo of C_2 & C_3 must store same total charge.

$q_2 = C_2 \Delta V_{\parallel}$

$q_3 = C_3 \Delta V_{\parallel}$

since $C_3 \gg C_2$

$q_3 \gg q_2$

$\therefore q_3$ increase (1)

q_2 decrease (1)